Technical Comments

Comments on "Analytical Design of Optimal Nutation Dampers"

PHUNG K. NGUYEN*
Institute for Aerospace Studies,
University of Toronto, Toronto, Ontario, Canada

THE problem of minimizing the decay time of a special fourth-order system has been recently analyzed by Amieux and Dureigne.[1] Unfortunately, their "optimal" solution is not, in fact, optimal.

Let the fourth-order system be of the form

$$\lambda^4 + \Pi A_1 \lambda^3 + A_2 \lambda^2 + \Pi A_3 \lambda + A_4 = 0$$

where $\Pi(>0)$ is a free parameter and A_i $(i = 1, \ldots, 4)$ are positive constants. Then it can be proved that the optimal root distribution should assume one of the ten configurations in Figs. 1a and 1b. The definition of "optimal" is as stated by Amieux and Dureigne.[1] The coefficients a_i^* in Figs. 1a and 1b are defined by

$$a_1^* = A_1\Pi - 4\delta$$
$$a_2^* = A_2 - 3\delta A_1 \Pi + 6\delta^2$$
$$a_3^* = A_3\Pi - 2\delta A_2 + 3\delta^2 A_1 \Pi - 4\delta^3$$
$$a_4^* = A_4 - \delta A_3 \Pi + \delta^2 A_2 - \delta^3 A_1 \Pi + \delta^4$$

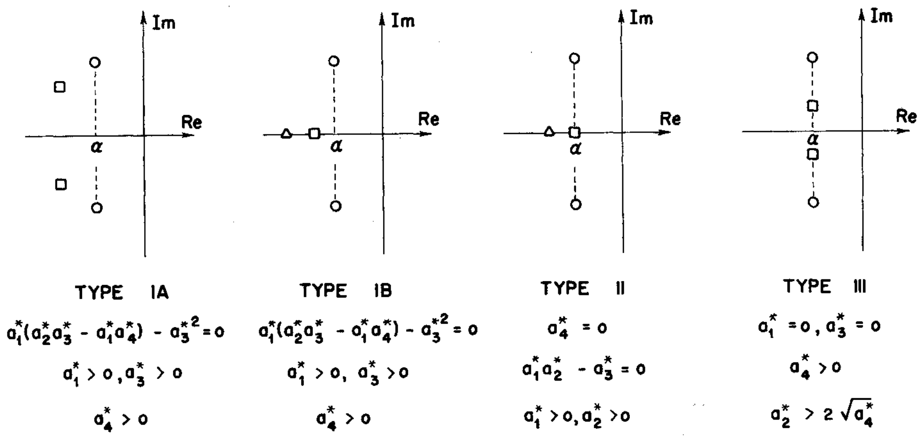

Fig. 1a Distributions of single roots under optimal condition.

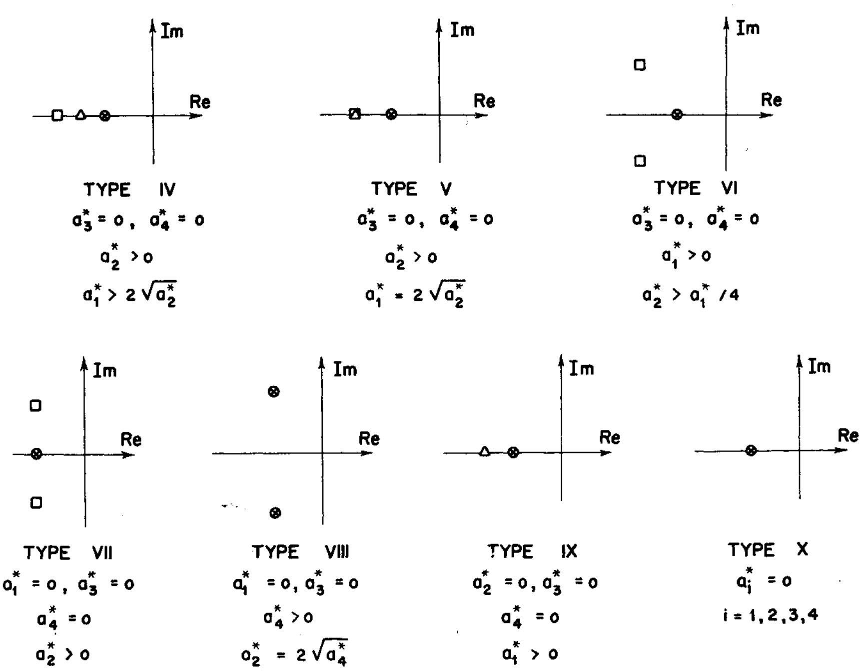

Fig. 1b Distributions of multiple roots under optimal condition.

Received March, 1973.
Index category: Spacecraft Attitude Dynamics and Control.
* Graduate Student.

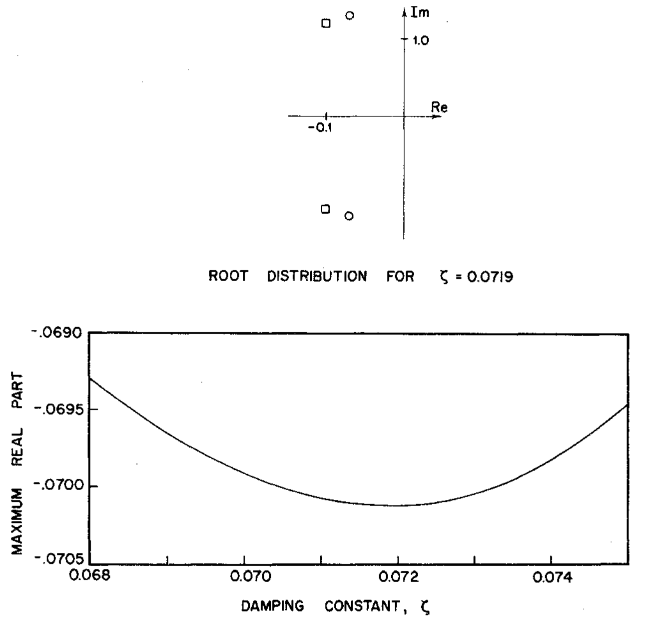

Fig. 2 Behavior of maximum real part in the neighborhood of optimal solution.

where δ is a positive constant. Details on these root distributions are discussed elsewhere.[3,4] Of these ten root configurations, the optimal condition used by Amieux and Dureigne,[1] namely $a_1^* = 0$, represents an a priori restriction to one of the four types (III, VII, VIII, X); the other possibilities are discarded at the outset. Their condition is therefore in error since unless there are further constraints on the A_i ($i = 1, \ldots, 4$), all types are possible.

On the other hand, suppose that the true optimal solution corresponds to one of the four types (III, VII, VIII, X). Then their analytical solutions can be derived (exactly) without difficulty. With a, b, c, d, e, ρ, r, ω^2 as defined by Amieux and Dureigne[1] and with $A_1 = (1+\rho b)/(1+\rho a)$, $A_2 = (r+\omega^2+\rho c)/(1+\rho a)$, $A_3 = (\omega^2+\rho d)/(1+\rho a)$, $A_4 = (\omega^2 r+\rho e)/(1+\rho a)$, then the results can be established as described in Table 1.

Table 1 Conditions on A_i ($i = 1, \ldots, 4$) for optimal solution q^*

Type of optimal solution	Optimal solution $= q^*$	Conditions on $A_i, i = 1, \ldots, 4$
Amieux and Dureigne	$q^* = 2[\rho(-a\omega^4+c\omega^2-e)]^{1/2}/\omega$	Nil
Type III	$q^* = 2[r-\omega^2+\rho(ar+br+c-2d+b\omega^2-3a\omega^2)+\rho^2(ab(r+\omega^2)+c(a+b)-2a^2\omega^2-4ad)+\rho^3(abc-2a^2d)]/(1+\rho b)^{3/2}$	$2A_3 < A_1A_2 < 6A_3$ $16A_1^3A_4+A_1^2A_2^2-12A_1A_2A_3+20A_3^2 = 0$ $A_1^2A_2^2+2A_3^2-2A_1A_2A_3-2A_4A_1^2 > 0$
Type VII	Same as Type III	$2A_3 < A_1A_2 < 6A_3$ $16A_1^3A_4+A_1^2A_2^2-12A_1A_2A_3+20A_3^2 = 0$
Type VIII	$q^* = 2[1+\rho a]\cdot[(1+\rho b)^2\cdot(\omega^2(1-r)+\rho(d-e)-(1+\rho a)\cdot(\omega^4+\rho^2d^2+2\rho d\omega^2)]/[(1+\rho b)^{3/2}\cdot(\omega^2+\rho d)^{1/2}]$	$A_3-A_4-A_3^2/A_1^2 > 0$ $A_3^2-A_1A_2A_3+A_1^2(A_3-A_4) = 0$
Type X	$q^* = 4(1+\rho a)\cdot(\omega^2+\rho d)^{1/2}/(1+\rho b)^{3/2}$	$A_1A_2-6A_3 = 0$ $A_1^2A_3-A_4A_1^2-A_3^2 = 2A_1A_3$

It can be seen that none of the exact solutions reduces to Amieux and Dureigne's[1] result. The discrepancy indeed comes from the following three approximations: 1) the characteristic equation is approximated by a truncated Taylor series; 2) the real part of the approximated characteristic equation is again approximated; and 3) the imaginary part of the approximated characteristic equation is not satisfied by the solution to 2).

The discrepancy can be illustrated by a numerical example of their ball-in-tube system. For $m = 0.150$ Kg, $\omega_3 = 2\Pi$ rad/sec, $\omega = 0.4\Pi$ rad/sec, $I_1 = 33/1.2$ m^{2}Kg, $l = 0.1730$ m, $g = 0.04$, the Amieux and Dureigne's optimal solution is $\zeta^* = 0.07897$ (from $\zeta^* = 2.8\ m|\omega-\omega_3|[\rho(\omega+\omega_3)/\omega]^{1/2}$) with 4 roots having the same real part -0.094035.

The true optimal solution is found to be of type I and the variation of the maximum real part with ζ is shown in Fig. 2. This proves that a wrong assumption on root distribution was used. The indirect optimization method proposed by Hughes[2,5] gives the optimal solution $\zeta^* = 0.07191$ and the maximum real part $= -0.070129$ as the solution to $T_2 = \partial T_2/\partial\zeta = 0$, where $T_2 = a_1^*a_2^*a_3^* - a_3^{*2} - a_1^{*2}a_4^*$.

References

[1] Amieux, J. C. and Dureigne, M., "Analytical Design of Optimal Nutation Dampers," *Journal of Spacecraft and Rockets*, Vol. 9, No. 12, Dec. 1972, pp. 934–935.

[2] Hughes, P. C. and Nguyen, P. K., "Minimum Time Decay in Linear Stationary Systems," *IEEE Transactions on Circuit Theory*, Vol. CT-19, No. 2, March 1972.

[3] Borrelli, R. L. and Leliakov, I. P., "An Optimization Technique for the Transient Response of Passively Stable Satellites," *Journal of Optimization Theory and Applications*, Vol. 10, No. 6, 1972, pp. 344–361.

[4] Nguyen, P. K., "A Practical Method for Optimum Transient Riddance in Damped Linear System," TN 151, 1970. Institute for Aerospace Studies, University of Toronto, Toronto, Ontario, Canada.

[5] Hughes, P. C., "Optimum Transient Riddance in Damped Linear System," *CASI Transactions*, Vol. 1, No. 1, 1968, pp. 14–20.

Reply by Author to P. K. Nguyen

J. C. Amieux*
NASA Goddard Space Flight Center, Greenbelt, Md.

P. K. NGUYEN brought to our attention the outstanding study done by R. L. Borrelli and I. P. Leliakov[1] published at the same time as our paper,[2] unknown to the authors. For the class of dynamical systems defined in Ref. 1, the problem is now completely solved. P. C. Hughes and P. K. Nguyen[3] utilize the variational approach to minimize the real part of the least damped root of the system. Let us compare the results of our approach and P. K. Nguyen's approach with Borrelli and

Received October 11, 1973.
Index category: Spacecraft Attitude Dynamics and Control.
* ESRO Fellow.